Computing TIE Factors for Non-telecom Applications

Contents

1 Introduction

Time-interval error (TIE) is defined as the short-term variations of the significant instants of a digital signal from their ideal positions in time [1-3]. The methodologies discussed on how to compute crest factors for TIE in this application note fall into two groups: methodologies for (1) telecommunications (i.e., "telecom") applications, and (2) non-telecom applications. Telecom applications generally revolve around a narrow group of industry standards including SONET, SDH, and OTN. These standards quantify total jitter as RMS and peak-peak values based on analog measurements taken within a 60-second time interval. Non-telecom applications (are assumed here to) include everything else, and are associated with a wide variety of industry standards (e.g., Fibre-channel, PCI Express, Ethernet, etc.). These standards decompose total jitter into random and deterministic components to estimate total jitter at a low target bit-error ratio (BER). This document addresses non-telecom applications. Refer to application note *AN10071 Computing TIE Crest Factors for Telecom* [4] for a discussion of telecom applications.

Any measurement of jitter results in a total jitter (TJ) value. This TJ value may be decomposed into both random and deterministic components of jitter. The industry refers to the random component of TJ as random jitter (RJ), and the deterministic component of TJ as deterministic jitter (DJ).

The TIE crest factor discussed in this document relates only to RJ. Major sources of RJ in a system include oscillator noise and (in optical systems) photodetector noise. RJ is typically modeled as a zero-mean Gaussian distribution, also called a normal distribution, as shown in Figure 1.1, where σ is the standard deviation of the distribution. Note that σ is equivalent to the RMS value for this distribution since the distribution's mean is zero.

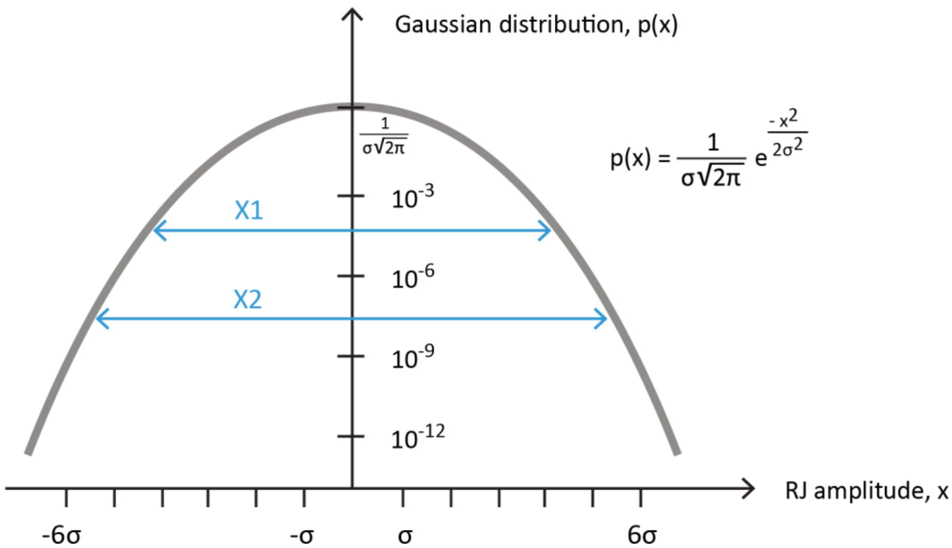

Figure 1.1: A Gaussian distribution plotted on a logarithmic scale

The probability of measuring larger peak-peak RJ values increases with measurement time. For example, suppose that for given a time T1, a maximum peak-peak RJ value of X1 is measured. If the measurement time increases to T2, a maximum peak-peak RJ value of X2 may be measured, where X2≥X1, as shown in Figure 1.1.

The crest factor N is defined (for the purposes of this document) as the ratio of peak-peak to RMS values, or

N = peak-peak value ÷ RMS value

The crest factor may be computed for any signal. For example, the crest factor for a sine wave is $2\sqrt{2}$. The crest factors for X1 and X2 shown in Figure 1.1 are X1 ÷ σ and X2 ÷ σ, respectively.

Regarding RJ for non-telecom applications, the crest factor N specifies how many standard deviations into the RJ Gaussian tail to include when converting RJ RMS (i.e., σ) to RJ peak-peak. The following sections discuss how to compute the crest factor and use it to estimate the contribution of RJ to TJ at a target BER using the dual-Dirac model given by,

TJ(target BER) = DJδδ + N(target BER) × σ

where DJδδ is the dual-Dirac deterministic jitter (which consists of a pair of delta functions separated by a distance of DJδδ) [5].

2 Computing Crest Factors from a Bathtub Curve

Figure 2.1 shows an example receiver in a serial-data communications link. The receiver recovers a bit clock from a data stream and uses it to retime the data at a flip flop. The flip flop's decision is analyzed in Figure 2.1 assuming only RJ in the data signal and no jitter in the bit clock. The RJ in the data signal is drawn (not to scale) as a Gaussian distribution. This distribution appears at each edge in the data signal, of which two edges (i.e., a right and a left data edge) are drawn in Figure 2.1.

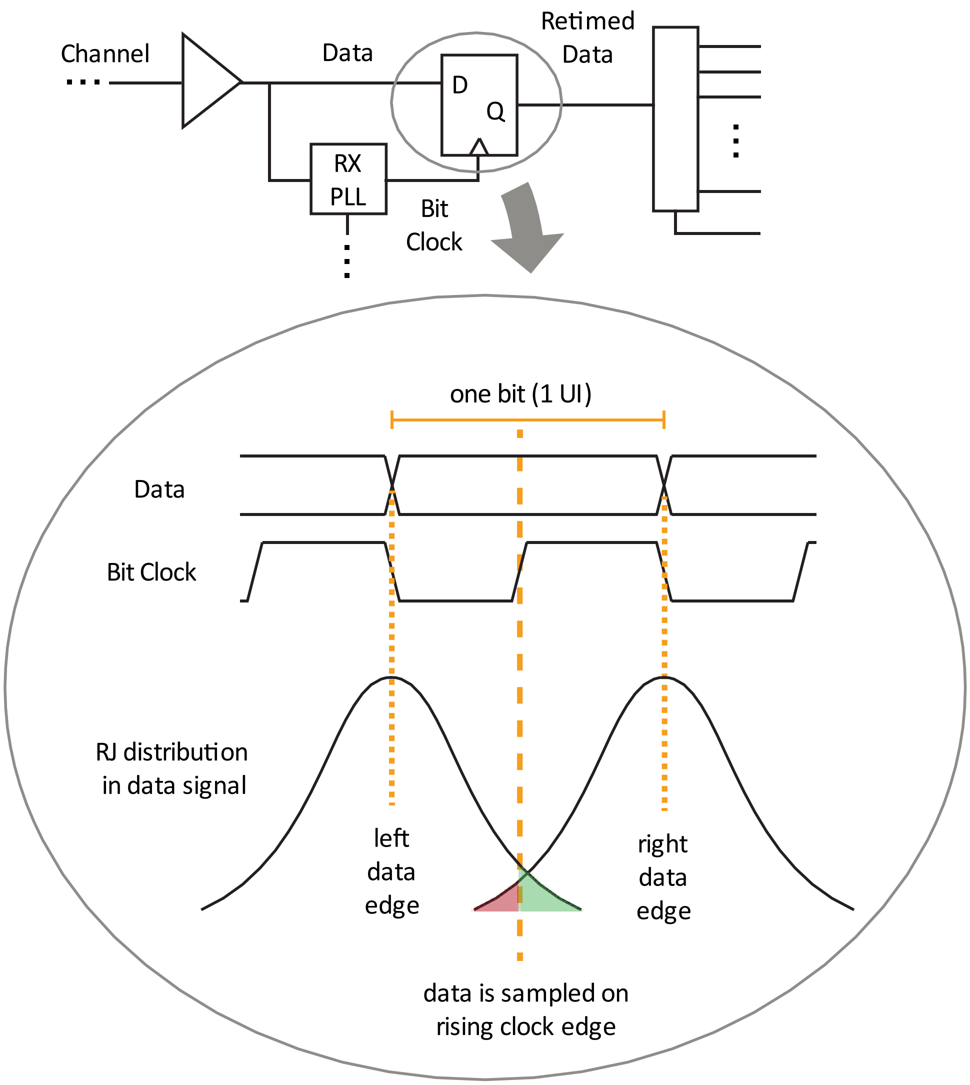

Figure 2.1: Analysis of bit errors due to RJ in an example serial-link receiver

If the bit clock's rising edge samples the data, a bit error is seen to occur if (1) the left data edge arrives late (i.e., to the right of the sampling clock edge), or (2) the right data edge arrives early (i.e., to the left of the sampling clock edge). The system BER is proportional to the sum of these two errors. The probability that the data's left edge arrives late equals the green-shaded region of the left data edge's distribution divided by the total area of the distribution. The probability of error from the right data edge is computed similarly using the red-shaded region of the right data edge's distribution. Figure 2.1 is arbitrarily drawn with the sampling clock edge closer to the left data edge than the right data edge. Therefore, the green-shaded region is larger than the red-shaded region, since more errors are expected from the left data edge than the right data edge.

Let's first analyze errors from the left data edge arriving late. The RJ distribution is typically modeled [5] as a probability density function (PDF) having a zero-mean Gaussian distribution, or,

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \quad \text{Eq. 1}$$

where σ is the standard deviation of the distribution. The left data edge's distribution from Figure 2.1 is redrawn in Figure 2.2, where the sampling clock's rising edge is located a distance x = Qσ to the right of the mean location of the left data edge. The shaded area in Figure 2.2 therefore represents bit errors resulting from the clock sampling the data before the data has settled (i.e., before the left data edge transitions).

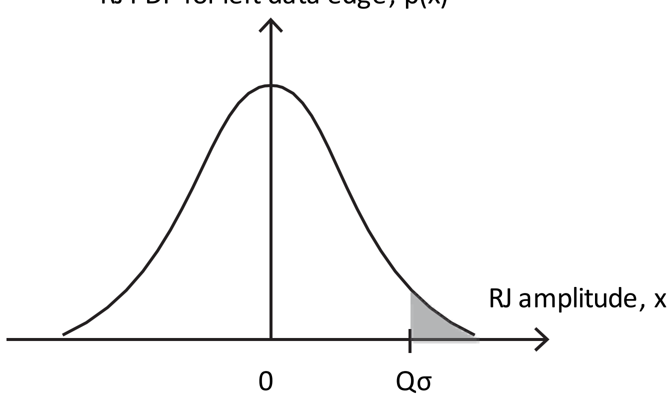

Figure 2.2: The probability that the left data edge arrives late equals the shaded area divided by the total area of the distribution

The probability of a bit error (from the left edge) equals the probability that RJ causes the data to transition at, or to the right of, the sampling clock transition at x = Qσ, which may be computed as,

$$BER_{Left}(Q) = P(X \geq Q\sigma) = \int_{Q\sigma}^{\infty} p(x)dx \quad \text{Eq. 2}$$

Substituting p(x) from Eq. 1 gives,

$$BER_{Left}(Q) = \frac{1}{\sigma\sqrt{2\pi}} \int_{Q\sigma}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx \quad \text{Eq. 3}$$

Performing a change of variable t, where

$$t = \frac{x}{\sqrt{2}\sigma} \qquad \text{Eq. 4}$$

simplifies the equation to,

$$BER_{Left}(Q) = \frac{1}{\sqrt{\pi}} \int_{\frac{Q}{\sqrt{2}}}^{\infty} e^{-t^2} dt \qquad \text{Eq. 5}$$

which can be rewritten using the complementary error function,

$$erfc(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-t^2} dt \qquad \text{Eq. 6}$$

as,

$$BER_{Left}(Q) = 0.5 \times erfc\left(\frac{Q}{\sqrt{2}}\right) \qquad \text{Eq. 7}$$

where erfc(z) may be computed from a variety of sources including Microsoft Excel.

The discussion so far assumes that the left data edge transition always occurs. Therefore, Eq. 7 is only valid for data signals having a "1010" clock-like data pattern. Since a bit error can *only* occur if a data transition exists, the BER should improve (i.e., reduce) for data patterns having fewer transitions. To account for times when the data does not transition, Eq. 7 is modified as,

$$BER_{Left}(Q) = 0.5 \times DTD \times erfc\left(\frac{Q}{\sqrt{2}}\right) \qquad \text{Eq. 8}$$

where the data-transition density (DTD) is defined as the number of data-edge transitions divided by the number of data bits. When DTD information is not available, a value of 0.5 is typically assumed (and accurate for PRBS and 8B/10B encoded data streams).

Eq. 8 computes the BER caused by the left data edge arriving late, considering only RJ, where the RJ has a standard deviation of σ, and where the sampling clock samples the data at Q standard deviations to the right of the mean location of the left data edge (i.e., the sampling clock samples the data at Q standard deviations into the tail of the RJ distribution).

Using symmetry, a similar argument can be made to derive $BER_{Right}(Q)$ due to the right data edge arriving early. A bathtub plot, shown in Figure 2.3, is created by plotting curves for $BER_{Left}(Q)$ and $BER_{Right}(Q)$ over one unit interval (UI) (where 1 UI is the duration of one data bit). The bathtub plot traces out the BER resulting from a sampling clock's edge stepping through one bit of data (from the left data edge at 0 UI to the right data edge at 1 UI). From Figure 2.3, the total eye closure at a target BER is observed as 2Qσ.

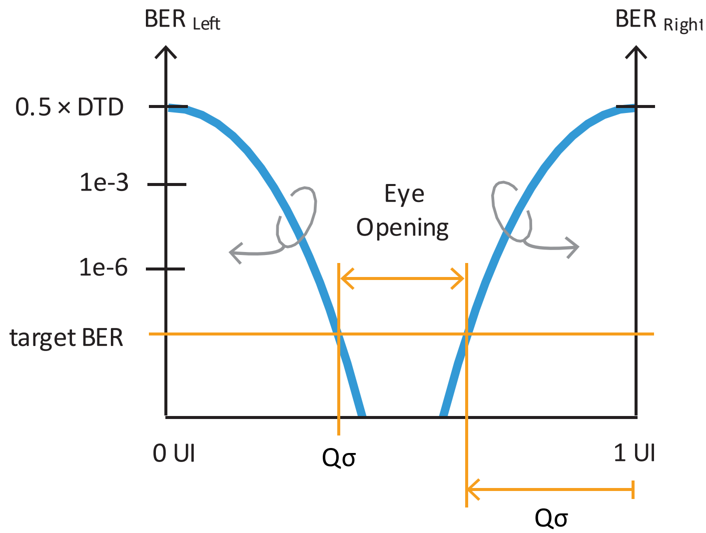

Figure 2.3: Bathtub curve created by plotting curves for $BER_{Left}(Q)$ and $BER_{Right}(Q)$

In practice, a target BER is known (such as from an industry standard), and the value for σ is independently measured. Eq. 8 is used to solve for Q, and the RJ RMS value (i.e., σ) converted to peak-peak using,

RJ peak-peak = 2Qσ

By definition, the crest factor N is the ratio of peak-peak to RMS, and equals,

N = RJ peak-peak ÷ σ = 2Q

Substituting N in Eq. 8 and solving for a target BER gives,

$$target\ BER = 0.5 \times DTD \times erfc\left(\frac{N}{\sqrt{8}}\right) \qquad \text{Eq. 9}$$

Table 1.1 summarizes crest factors computed from Eq. 9 for a range of target BER and two common DTD values. This table is typically used to compute the contribution of RJ to TJ at a given BER.

For example, suppose the data signal shown in Figure 2.1 is a PRBS pattern (i.e., DTD = 0.5) where each data edge has a TIE RJ RMS value of σ. If the application is, for example, 10Gb Ethernet, then the target BER is 1e-12. The crest factor N may be obtained from the above table (or Eq. 9) to be 13.874, and multiplied by σ to obtain the total peak-peak eye closure (at the target BER) caused by RJ.

One caveat in the above analysis is an assumption that the amplitude of jitter in the signal is sufficiently small such that $BER_{Left} >> BER_{Right}$ where the target BER intersects the BER_{Left} curve (and likewise, $BER_{Right} >> BER_{Left}$ where the target BER intersects the BER_{Right} curve). This is generally a good assumption (and is built into many standards).

3 Computing Crest Factors when DJ Splits the Distribution

Some industry standards adopt a methodology advanced by the INCITS Fibre Channel T11 committee, author of the popular MJSQ [6] document, which assumes the dual-Dirac model splits the RJ distribution into two independent halves. This requires the DJ component to be large enough that the left side of the distribution has no effect on the right side of the distribution (and vice versa). Figure 3.1 illustrates this process, where the blue curves (drawn for reference) correspond to the original RJ distribution discussed in the last section. An example is shown where this RJ distribution is convolved with a DJ distribution (e.g., modeled using dual-Dirac) to create a total jitter PDF (top graph) and corresponding BER_{Left} (bottom graph) curves in red. The red PDF curve (top graph) illustrates the splitting of the original blue PDF into two halves, whose effects may be treated independently from the point of view of computing BER_{Left}.

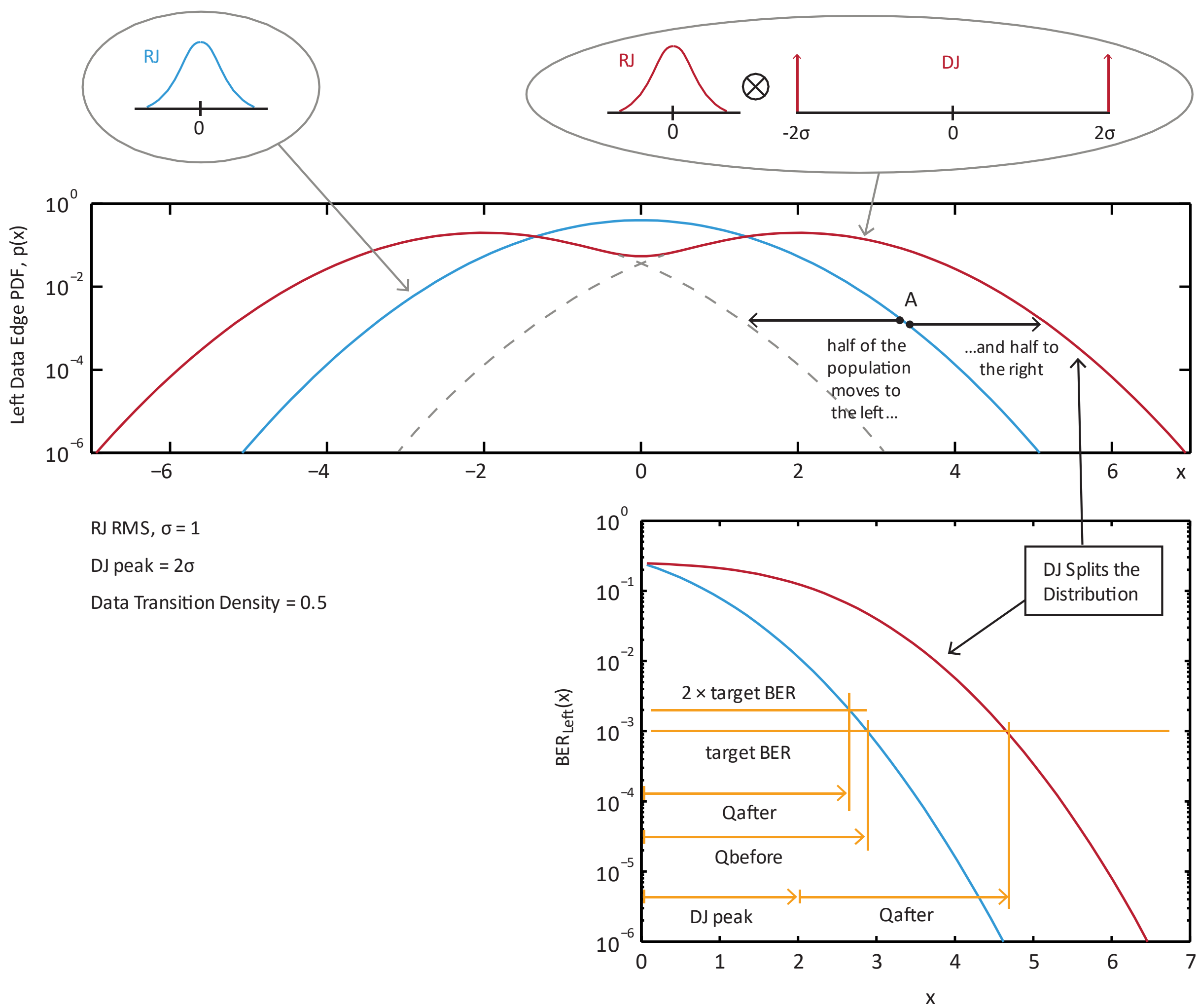

Figure 3.1: Plots of PDF(x) (top graph) and BER_{Left}(x) (bottom graph) for a RJ distribution (blue curves) and a distribution resulting from the convolution of RJ and DJ that splits the original distribution into two halves in which only the near one effects the error rate (red curves). The mean location of the left data edge is at x = 0 in the above plots.

The blue BER_{Left} curve (for the original RJ distribution) may be computed using Eq. 8, where Q is varied from 0 to 7. The Qbefore value shown in Figure 3.1 is obtained by setting Eq. 8 to the target BER (e.g., 1e-3 here) and solving for Q.

After the distribution splits, the dual-Dirac model is still desired to estimate TJ at a low target BER. Figure 3.1 shows DJδδ is a peak-peak value equal to 2 × (DJ peak) = 4σ, and the crest factor N is evaluated at a target BER of 2×Qafter. The question becomes, how to compute N? Note that the RJ RMS value of σ does not change when the distribution splits.

Rather than computing the red BER_{Left} curve then measuring Qafter, it's simpler to use the equations developed in the previous section with a little understanding. Analyzing the RJ PDF after it splits (ignoring DJ temporarily here) is equivalent to analyzing a new PDF that is exactly half of the original Gaussian distribution discussed in the last section (e.g., half of the blue PDF in Figure 3.1). In other words, when the PDF splits, half of the population in the original Gaussian distribution (e.g., blue PDF curve) at point A in Figure 3.1 moves to the right (landing on the right dual-Dirac half) and half moves to the left (landing on the left dual-Dirac half). It may be difficult to view, but the peak PDF height drops a factor of two in Figure 3.1 comparing before (blue) versus after (red) the split.

For the purposes of computing a crest factor, the effect of splitting the original Gaussian distribution to produce the red BER_{Left} curve is equivalent to sliding up the (original) blue BER_{Left} curve to twice the target BER and solving for Qafter using Eq. 8 (or, equivalently, N using Eq. 9), as illustrated in Figure 3.1. Thus, Eq. 9 is rewritten, where DJ is sufficiently large to split a Gaussian distribution, as,

$$2 \times BER_{Left}(Q) = DTD \times erfc\left(\frac{Q}{\sqrt{2}}\right) \qquad \text{Eq. 10}$$

$$2 \times target\ BER = DTD \times erfc\left(\frac{N}{\sqrt{8}}\right) \qquad \text{Eq. 11}$$

Table 1.2 summarizes crest factors computed from Eq. 11 for a range of target BER and two common DTD values.

4 Crest Factor Tables

Table 1.1 and Table 1.2 below summarize crest factors when the data's jitter distribution does not split and does split, respectively, into two independent halves, as discussed in the previous two sections.

Table 1.2: Eq. 9 Crest Factors (the distribution does not split)

target BER	Crest Factor (N = 2Q)	
	DTD = 0.5	DTD = 1
1e-1	1.683	2.563
1e-2	4.108	4.653
1e-3	5.756	6.180
1e-4	7.080	7.438
1e-5	8.215	8.530
1e-6	9.223	9.507
1e-7	10.138	10.399
1e-8	10.982	11.224
1e-9	11.768	11.996
1e-10	12.508	12.723
1e-11	13.208	13.412
1e-12	13.874	14.069
1e-13	14.511	14.698
1e-14	15.122	15.301
1e-15	15.710	15.883
1e-16	16.277	16.444

Table 1.1: Eq. 11 Crest Factors (the distribution splits)

target BER	Crest Factor (N = 2Q)	
	DTD = 0.5	DTD = 1
1e-1	0.507	1.683
1e-2	3.501	4.108
1e-3	5.304	5.756
1e-4	6.706	7.080
1e-5	7.889	8.215
1e-6	8.930	9.223
1e-7	9.871	10.138
1e-8	10.734	10.982
1e-9	11.537	11.768
1e-10	12.290	12.508
1e-11	13.001	13.208
1e-12	13.677	13.874
1e-13	14.322	14.511
1e-14	14.941	15.122
1e-15	15.535	15.710
1e-16	16.108	16.277

5 Conclusion

This note presented two common methods for computing TIE crest factors. Eq. 10 (Table 1.2) should be used when only one half of the dual-Dirac distribution contributes to errors in the bathtub curve (at the location where the target BER is evaluated). Otherwise, if both halves contribute to errors, for example (1) if there is no DJ, or (2) if there is a lot of jitter (random, deterministic, or both) centered at the mean location of the data edge, then Eq. 9 (Table 1.2) is more accurate. When in doubt, assuming the distribution does not split (Eq. 9) provides a more pessimistic result. At a target BER of 1e-12, the difference is less than 2%.

Finally, keep in mind that it is the data signal's jitter distribution (see Figure 2.1) that must be evaluated to determine which method to compute the crest factor. Of course, clock jitter can still play an important role. For example, suppose one wishes to estimate how much a clock device contributes to an application's TJ budget for data, where the clock is used to retime the data. Here, one may multiply the clock device's phase jitter RMS value (e.g., TIE RJ RMS value) by a crest factor obtained from Eq. 11 (Table 1.2), even if the clock device only has random jitter, if the customer's application is known (or defined by an industry standard) to have sufficient DJ such that the data's jitter distribution (after being timed by the clock) splits as described above.

6 References

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[2] "Synchronous Optical Network (SONET) Transport Systems: Common Generic Criteria," Telcordia, GR-253-CORE, Issue 4 (December 2005), section 2.2.4.

[3] Application Note, "AN10007 Clock Jitter Definitions and Measurement Methods," SiTime, https://www.sitime.com/support/resource-library/an10007-clock-jitter-definitions-and-measurement-methods

[4] Application Note, "AN10071 Computing TIE Crest Factors for Telecom Applications," SiTime, https://www.sitime.com/support/resource-library/application-notes/an10071-computing-tie-crest-factors-telecom-applications

[5] "Jitter Analysis: The dual-Dirac Model, RJ/DJ, and Q-Scale," White Paper by Ransom Stephens, Agilent Technologies (December 31, 2004), http://cp.literature.agilent.com/litweb/pdf/5989-3206EN.pdf

[6] "Methodologies for Jitter and Signal Quality Specification (MJSQ)," INCITS, Fibre Channel, T11.2 / Project 1316-DT/ Rev 14.1 (June 2005).

Table 2: Revision History

Version	Release Date	Change Summary
1.0	22-Jan-2014	JitterLabs LLC Initial Release
1.0	30-Mar-2021	SiTime Initial Release

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