Computing TIE Crest Factors for Telecom Applications

Contents

1 Introduction

Time-interval error (TIE) is defined as the short-term variations of the significant instants of a digital signal from their ideal positions in time [1-3]. The methodologies discussed on how to compute crest factors for TIE in this application note fall into two groups: methodologies for (1) telecommunications (i.e., "telecom") applications, and (2) non-telecom applications. Telecom applications generally revolve around a narrow group of industry standards including SONET, SDH, and OTN. These standards quantify total jitter as RMS and peak-peak values based on analog measurements taken within a 60-second time interval. Non-telecom applications (are assumed here to) include everything else, and are associated with a wide variety of industry standards (e.g., Fibre-channel, PCI Express, Ethernet, etc.). These standards decompose total jitter into random and deterministic components to estimate total jitter at a low target bit-error ratio. This document addresses telecom applications. Refer to application note *AN10072 Computing TIE Crest Factors for Non-telecom Applications* [4] for a discussion of non-telecom applications.

Any measurement of jitter results in a total jitter (TJ) value. This TJ value may be decomposed into both random and deterministic components of jitter. The industry refers to the random component of TJ as random jitter (RJ), and the deterministic component of TJ as deterministic jitter (DJ).

The TIE crest factor discussed in this document relates only to RJ. Major sources of RJ in a system include oscillator noise and (in optical systems) photodetector noise. RJ is typically modeled as a zero-mean Gaussian distribution as shown in Figure 1.1, where σ is the standard deviation of the distribution. Note that σ is equivalent to the RMS value for this distribution since the distribution's mean is zero.
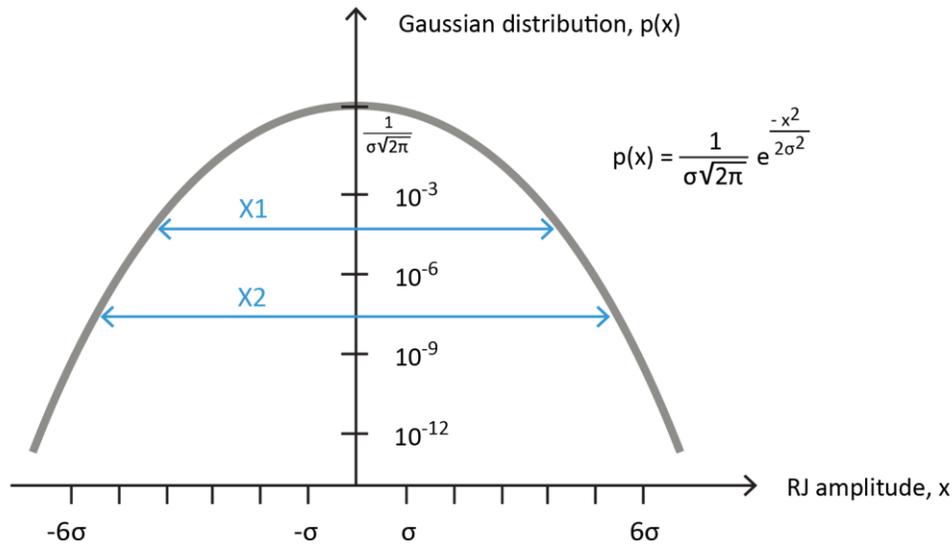
Gaussian distribution, p(x)

$\frac{1}{\sigma\sqrt{2\pi}}$

$p(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \ e^{\frac{-x^2}{2\sigma^2}}$

10^{-3}

10^{-6}

10^{-9}

10^{-12}

X1

X2

RJ amplitude, x

-6σ   -σ   σ   6σ

Figure 1.1: A Gaussian distribution plotted on a logarithmic scale

The probability of measuring larger peak-peak RJ values increases with measurement time. For example, suppose that for given a time T1, a maximum peak-peak RJ value of X1 is measured. If the measurement time increases to T2, a maximum peak-peak RJ value of X2 may be measured, where X2≥X1, as shown in Figure 1.1.

The crest factor N is defined (for the purposes of this document) as the ratio of peak-peak to RMS values, or

> N = peak-peak value ÷ RMS value

The crest factor may be computed for any signal. For example, the crest factor for a sine wave is $2\sqrt{2}$. The crest factors for X1 and X2 shown in Figure 1.1 are X1 ÷ σ and X2 ÷ σ, respectively.

Regarding RJ for telecom applications, the crest factor N specifies how many standard deviations into the RJ Gaussian tail may be observed in a specified time interval. Since telecom industry standards [2] require total jitter to be measured for 60 seconds, this time interval is the main focus of this document.

The following discussion derives a value for the crest factor to estimate the contribution of RJ to TJ over a specified time interval. The following procedure is adopted from [5].

2   Procedure to Compute Crest Factors

Telecom jitter measurements are traditionally measurements of phase noise [6-7]. Phase-noise refers to phase deviations in a clock signal, and results from random phase fluctuations disrupting a periodic signal's linear phase trend. Phase noise is characterized as a power spectral density of phase fluctuations, and assigned units of decibels below the carrier in a 1-Hz bandwidth (i.e., dBc/Hz).

As shown in Figure 2.1, a telecom jitter measurement (similar to an absolute phase-noise measurement) is performed by connecting a device under test (DUT) and a reference source to a quadrature phase detector. The phase detector mixes these two signals to output sum and difference products. A phase-locked loop controls the frequency output of the reference source to establish a 90-degree phase difference between the DUT and the reference source. A low-pass filter removes the sum product output of the phase detector, passing only the difference product to the low-noise amplifier (LNA). The LNA improves the noise floor of the baseband analysis hardware by amplifying the baseband signal. For telecom applications, the baseband analysis hardware includes peak and RMS detectors to measure the maximum peak-peak and RMS total jitter values characterizing the baseband signal, which may then be used to determine compliance.
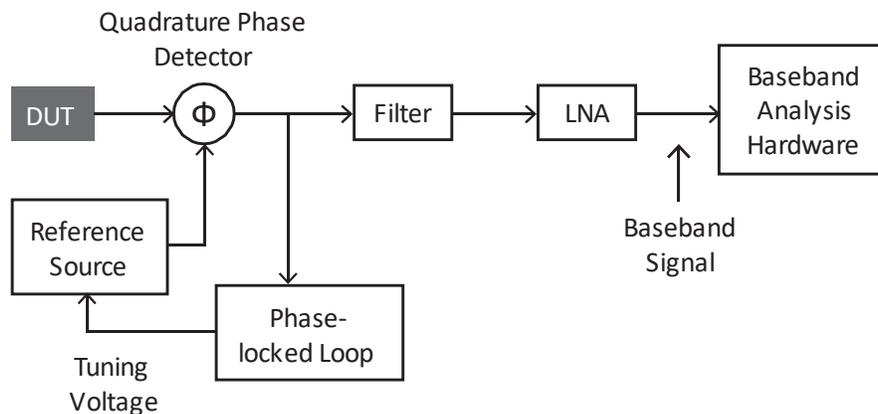

Figure 2.1: Simplified block diagram of telecom jitter measurement

The baseband signal is a continuous waveform of phase error in the time domain, which when transformed into the frequency domain, is phase noise. Integrating the phase-noise spectrum (over a bandwidth specified by a telecom standard) produces a phase jitter value in seconds RMS. This phase jitter value estimates the RJ component of TIE total jitter, and is used as the standard deviation σ in a Gaussian model for RJ. As shown in Figure 1.1, simply knowing σ is sufficient to model the entire RJ distribution. The question becomes, how deep (i.e., Nσ) into the Gaussian tail may be observed in a given time interval?

The answer lies in a procedure documented by [5]. Let the baseband-signal time function equal x(t), the measurement time interval equal T, and the bandwidth of the filter (which is a jitter filter having a low-pass cutoff frequency specified by telecom standards, for example [5]) equal B. Recall that the Nyquist sample rate equals twice the bandwidth of a band limited signal, which is 2B for function x(t). Therefore, if a signal x(t) having a bandwidth B is sampled at 2B over a time interval T, the minimum number of independent values measured is 2BT. Let n represent the population 2BT.

The random jitter is modeled as a zero-mean Gaussian probability distribution function P(x) with standard deviation of σ, as shown in Figure 2.2 where the Gaussian envelope is illustrated with a dashed line.
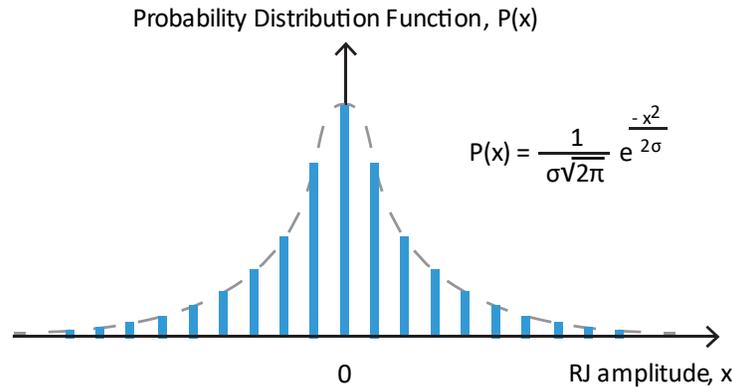
Probability Distribution Function, P(x)

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{\frac{-x^2}{2\sigma}}$$

0   RJ amplitude, x

Figure 2.2: RJ is modeled as a discrete zero-mean Gaussian distribution

The probability that RJ equals x is P(x), and therefore the sum of P(x) over all x equals 1. The probability that one measured value x_i of RJ is less than x equals the cumulative distribution function c(x),

$$c(x) \equiv Pr(-\infty \leq x_i \leq x) = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{x} e^{\frac{-t^2}{2}} dt \qquad \text{Eq. 1}$$

Since each measured RJ value is independent of all other measured RJ values, the probability that all measured values of RJ are less than x for a population of n independent measurements equals $c_{max}(x)$,

$$c_{max}(x) = \left(\frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{x} e^{\frac{-t^2}{2}} dt \right)^n \qquad \text{Eq. 2}$$

where $c_{max}(x)$ is referred to as the cumulative distribution function of the maximum RJ value. By definition, the probability distribution function $p_{max}(x)$ corresponding to $c_{max}(x)$ equals the derivative of $c_{max}(x)$,

$$p_{max}(x) = \frac{dc_{max}(x)}{dx} \qquad \text{Eq. 3}$$

The average, or expected, value of the maximum measured RJ is the integral of x multiplied by $p_{max}(x)$ over all x.

As an example, **Error! Reference source not found.** [5] illustrates these above steps where σ = 1 seconds RMS, T = 60 seconds, and BW = 80 MHz. The population n therefore equals 2BT = 9.6×10^9. The maximum expected measured RJ value is computed from $p_{max}(x)$ as 6.44 (using the code shown in the Appendix).

The minimum expected measured RJ value may be computed similarly. Assuming the distribution P(x) is symmetrical around its mean, the expected minimum is also 6.44. Therefore, the expected maximum RJ peak-peak value is 12.88, which is the crest factor N.
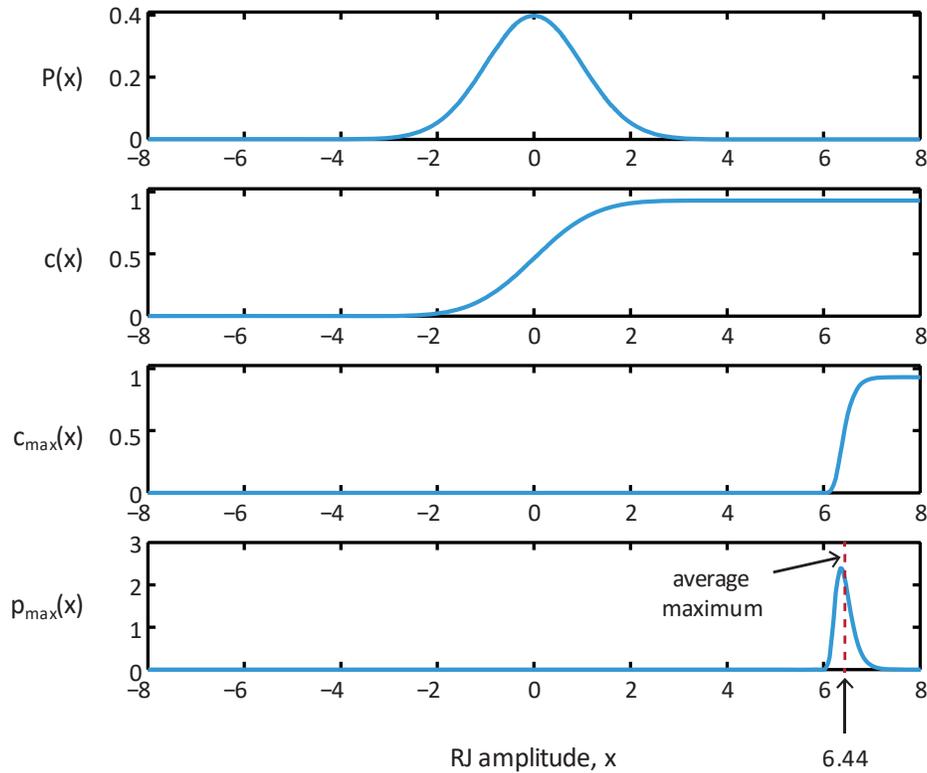

Figure 2.3: Summary of steps to compute a crest factor N for the expected maximum measured RJ value, where σ = 1 seconds RMS, T = 60 seconds, and BW = 80 MHz [5]

The ITU-T analysis [5] presented in Figure 2.3 for computing a crest factor is performed for a range of filter bandwidths, leading to the plot shown in Figure 2.4.
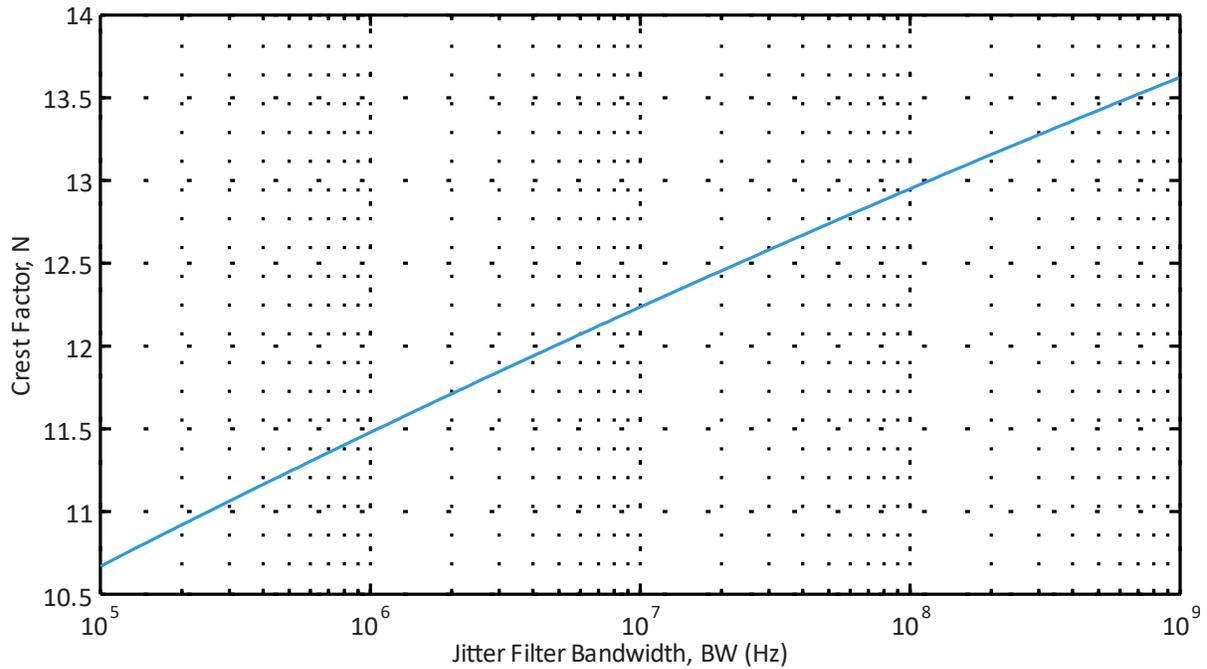
Figure 2.4: Plot of crest factor N versus jitter filter bandwidth BW

Telecom standards specify the filter bandwidth to use for jitter generation measurements. Some common bandwidths [8] are listed in Table 1.1, along with their corresponding crest factors computed using the procedure outlined in Figure 2.3.

Table 1.1: List of crest factors for common filter bandwidths

Filter BW (Hz)	Crest Factor (N = 2Q)
400 kHz	11.16
1.3 MHz	11.57
5 MHz	12.01
20 MHz	12.45
80 MHz	12.88
320 MHz	13.29

3   Conclusion

This note discussed a procedure [5] used in the telecom industry to compute crest factors. The crest factor may be used to compute the contribution of RJ to TJ over a specified time interval, such as 60 seconds. For example, if the value of phase jitter (in seconds RMS) is known for a device, this quantity may be converted to peak-peak seconds by multiplying the appropriate crest factor from Table 1.1 by the phase jitter value. This peak-peak seconds value estimates the maximum RJ peak-peak value expected for the device when measuring phase error (i.e., jitter) for 60 seconds.

4   References

[1] "Understanding and Characterizing Timing Jitter," application note, Tektronix (2003) www.tek.com

[2] "Synchronous Optical Network (SONET) Transport Systems: Common Generic Criteria," Telcordia, GR-253-CORE, Issue 4 (December 2005), section 2.2.4.

[3] Application Note, "AN10007 Clock Jitter Definitions and Measurement Methods," SiTime, https://www.sitime.com/support/resource-library/an10007-clock-jitter-definitions-and-measurement-methods

[4] Application Note, "AN10070 Computing TIE Crest Factors for Non-telecom Applications," SiTime, https://www.sitime.com/support/resource-library/application-notes/an10070-computing-tie-crest-factors-non-telecom

[5] ITU-T Recommendation O.172, "Jitter and wander measuring equipment for digital systems which are based on the synchronous digital hierarchy (SDH)," (April 2005), Appendix VIII.4, pp. 50-51.

[6] ITU-T Recommendation O.172, "Jitter and wander measuring equipment for digital systems which are based on the synchronous digital hierarchy (SDH)," (April 2005).

[7] "Jitter Analysis of High-Speed Digital Systems," by C. M. Miller, and D. J. McQuate, Hewlett-Packard Journal (1995), pp. 49-56.

[8] "Synchronous Optical Network (SONET) Transport Systems: Common Generic Criteria," Telcordia, GR-253-CORE, Issue 4 (December 2005), section 5.6.2.3.6.

5 Appendix

Matlab Code to Compute Crest Factor

Note that when implementing the algorithm presented above in computer code, the number of discrete values of x in the P(x) distribution needs to be carefully chosen. If too few values of x are used, the shape of $p_{max}(x)$ becomes distorted, which introduces errors. If too many values of x are used, numerical errors are introduced from quantization effects arising from reaching the limits of 64 bit floating point math, which eventually causes the crest factor to decrease with increasing BW. This study used 10000 values for x in P(x) in order to obtain crest factors up to BW = 1 GHz. The code is as follows.

```matlab
%% Matlab R2012B code to compute crest factors for telecom applications
%  using a methodology from ITU T-REC-O.172-200504 standard, section VIII.4.
%  © Copyright JitterLabs 2014.

%% Initialize session

format long e;
close all;
format compact;  % minimizes blank output lines
clear;

T = 60;         % measurement time in seconds
samples = 1e4;  % number of discrete x values in PDF
getPlot = 1;    % set to 1 to create a 4-panel plot for BW=plotBW
plotBW=80e6;

%% Create BW array

BW_spacing=log10(1.07/1);
log10BW=(5:BW_spacing:9);
BW=10.^(log10BW);  % array of equally spaced points on log scale

userBW=[400e3, 1.3e6, 5e6, 10e6, 20e6, 40e6, 80e6, 320e6, 1e9]; % specify any
special points to add to BW

arrBW=sort([BW,userBW]); % add userBW values to BW then sort
clear BW userBW log10BW;

%% Create Gaussian PDF

x=linspace(-8,8,samples)';
PDF = (1/sqrt(2*pi))*exp(1).^(-1*(x.*x)/2); % PDF=(1/sqrt(2))e^(-x*x/2)

if (getPlot==1)
    figure(1);
    subplot(4,1,1);
    plot(x,PDF,'b-');
    ylabel('P(x)');
    ylim([0,0.41]);
```

```matlab
end

%% Create Gaussian CDF (e.g. integral of PDF)

CDF=zeros(length(PDF)-1,1);
for ii=1:length(PDF)-1
    CDF(ii)=0.5*(x(ii+1)-x(ii))*(PDF(ii+1)+PDF(ii));    % trapezoidal rule =
0.5*(x2-x1)*(y2+y1)
end
CDF=cumsum(CDF);

if (getPlot==1)
    subplot(4,1,2);
    plot(x(2:end),CDF,'b-');
    ylabel('c(x)');
    ylim([0,1.1]);
end

%% Loop over BW
arrCF=zeros(length(arrBW),1);
for bb=1:length(arrBW)

    n=2*arrBW(bb)*T;

    %% CDFmax = CDF raised to N
    CDFmax = CDF.^n;

    if (getPlot==1 && arrBW(bb)==plotBW)
        subplot(4,1,3);
        plot(x(2:end),CDFmax,'b-');
        ylabel('cmax(x)');
        ylim([0,1.1]);
    end

    %% PDFmax = derivative of CDFmax
    PDFmax=zeros(length(CDFmax)-1,1);
    for ii=1:length(CDFmax)-1
        PDFmax(ii)=(CDFmax(ii+1)-CDFmax(ii))/(x(ii+2)-x(ii+1));  % derivative
    end

    if (getPlot==1 && arrBW(bb)==plotBW)
        subplot(4,1,4 );
        plot(x(3:end),PDFmax,'b-');
        xlabel('x'); ylabel('pmax(x)');
        ylim([0,3]);
    end

    %% Q = average (expected) value of the maximum = integral of x * PDFmax
over all x
    val=zeros(length(PDFmax),1);
    for ii=1:length(PDFmax)
        val(ii)=x(ii+2)*PDFmax(ii); % this is value we'll integrate
```

```matlab
    end

    Q=zeros(length(PDFmax)-1,1);
    for ii=1:length(PDFmax)-1
        Q(ii)=0.5*(x(ii+3)-x(ii+2))*(val(ii+1)+val(ii));  % trapezoidal rule =
0.5*(x2-x1)*(y2+y1)
    end

    QQ=sum(sort(Q));

    %% crest factor = 2*Q
    CF=2*QQ;

    arrCF(bb)=CF;

end

%% Plot Crest Factor versus BW
figure(2);
semilogx(arrBW,arrCF,'b-');
grid on;
xlabel('BW');
ylabel('Crest Factor, N');
```

Table 2: Revision History

Version	Release Date	Change Summary
1.0	22-Jan-2014	JitterLabs LLC Initial Release
1.0	30-Mar-2021	SiTime Initial Release